Moriah College
בית ספר הר המוריה
CELEBRATING 80 YEARS 1943-2023 | 5703-5783

Student Number

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions
- Reading time - 10 minutes
- Working time - 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided in addition to this paper
- A multiple choice answer sheet is provided in addition to this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I - 10 marks (pages 2-6)
- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 7- 37)
- Attempt Questions 11-35
- Allow about 2 hours 45 minutes for this section

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1   Consider the bivariate data shown on the scatterplot below.

Which of the following values is the best estimate for Pearson's correlation coefficient for this data?

A. -0.9

B. -0.2

C. 0.2

D. 0.9

2   The graph of the function $y = f(x)$ is moved 3 units to the left.

Which of the following is the new function?
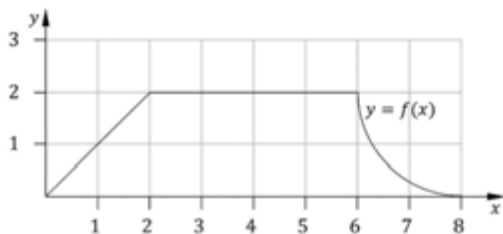
A. $y = f(x) + 3$

B. $y = f(x) - 3$

C. $y = f(x + 3)$

D. $y = f(x - 3)$

3 The height of the tide in a harbour can be modelled using the sine function. The time, t in hours, between high tide and low tide is 6 hours.

Which of the following could be the function representing the height of the tide?

A. $h = \sin\left(\frac{\pi t}{3}\right)$

B. $h = \sin\left(\frac{\pi t}{6}\right)$

C. $h = \sin\left(\frac{\pi t}{12}\right)$

D. $h = \sin\left(\frac{\pi t}{18}\right)$

4 The graph of $y = f(x)$ is shown.
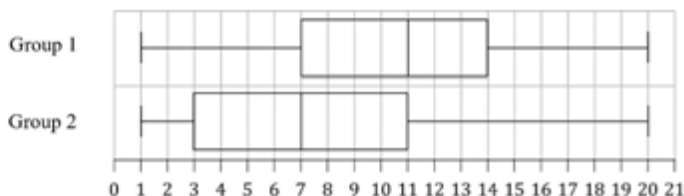

$y = f(x)$

DRAWN
TO
SCALE

What is the exact value of

$$\int_0^8 f(x)\, dx\,?$$

A. $10 + \pi$ units^2

B. $10 - \pi$ units^2

C. $14 + \pi$ units^2

D. $14 - \pi$ units^2

5 Nadav scored 34 in his Science examination. The mean of the scores was 50, and Nadav's z-score in the Science examination was -2.

What is the standard deviation of scores in the science examination?

A. 4

B. 8

C. 12
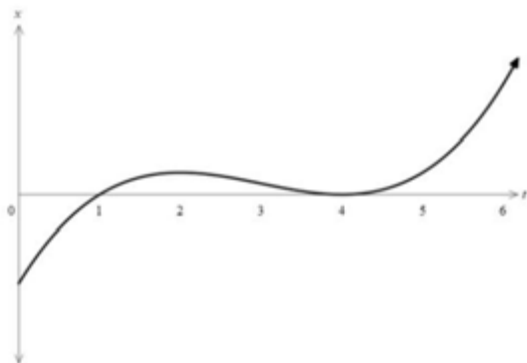
D. 16

6 Consider the parallel box plots below.

Which of the following statements is CORRECT?

A. Group 1 is positively skewed.

B. Group 2 is negatively skewed.

C. The difference between the median and Q_1 of Group 1 is the same as the difference between the median and Q_3 of Group 2.

D. The range and IQR are equal for both sets of data.

7

The displacement, x metres, from the origin of a particle moving in a straight line at any time, t seconds, is shown in the graph below.

When was the particle at rest?

A. $t = 0$

B. $t = 1, t = 4$

C. $t = 2, \ t = 4$

D. $t = 1, \ t = 2, \ t = 4$

8 A standard normal distribution has a mean of zero and a standard deviation of 1.

Given that $P(z_1 < Z < z_2) = 0.815$, which of the following are possible values of z_1 and z_2?

A. $z_1 = -1, z_2 = 2$

B. $z_1 = -1, z_2 = 3$

C. $z_1 = -2, z_2 = 2$

D. $z_1 = -2, z_2 = 3$

9 What is the derivative of $\left(\tan^2 x + 1\right)^2$?

A. $4\sec^3 x$

B. $4\sec^4 x$

C. $4\sec^3 x \tan x$

D. $4\sec^4 x \tan x$

10 It is given that $P(A) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.

What is the value of $P(\bar{B}|\bar{A})$?

A. $\frac{1}{4}$

B. $\frac{3}{10}$

C. $\frac{3}{4}$

D. $\frac{4}{5}$

End of multiple-choice section

Student Number

Mathematics Advanced
Section II Answer Booklet

Section II

90 marks
Attempt Questions 11-36
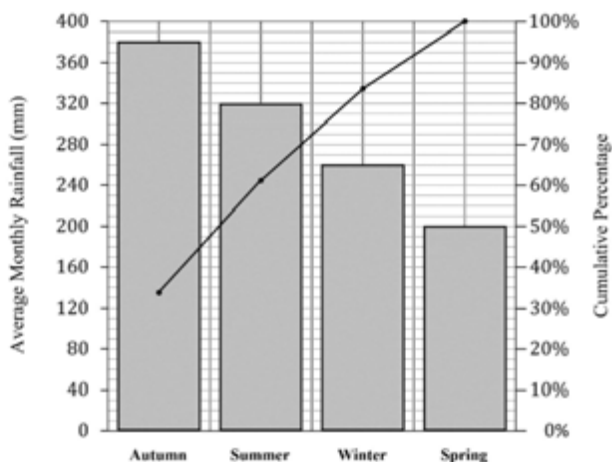Allow about 2 hours 45 minutes for this section

_____

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Please turn over

Question 11 (3 marks)

Below is a Pareto Chart showing the average seasonal rainfall in Sydney.

(a) What is the average rainfall for Summer?   1

(b) What percentage of the annual rainfall occurs in Winter and Spring together?   2

Question 12 (2 marks)

Find   **2**

$$\int \frac{1}{2x-3}\, dx\,.$$

Question 13 (3 marks)

Calculate and fully simplify: **3**

$$\int_{\ln 2}^{2\ln 2} e^{2x}\, dx\,.$$

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Question 14 (3 marks)

It is given that $f''(x) = 6x$ and that $f(x)$ has a stationary point at $(-1,3)$.　　**3**

Find $f(x)$.

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Question 15 (3 marks)

(a) It is given that $1 + 2x - 3x^2 \geq 0$ in the domain $[0,1]$.

    Prove that $f(x) = 1 + 2x - 3x^2$ is a probability density function for $[0,1]$.   **2**

(b) State the mode of the probability density function.   **1**

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Question 16 (4 marks)

Find and fully factorise the first and second derivatives of $y = x(x + 1)^3$.   **4**

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Question 17 (3 marks)

The circle $x^2 + y^2 - 6x + 8y - 11 = 0$ is transformed by a horizontal translation to the left by 4 units and a vertical translation up 3 units.

3

What is the centre and radius of the new circle?

Question 18 (3 marks)

Sianne owns a company producing and selling backpacks.
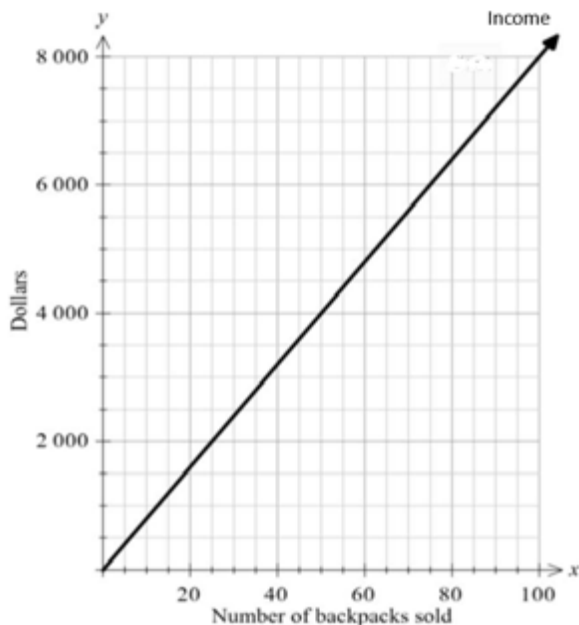The income is defined by the function $y = 80x$, where x is the number of backpacks sold.
The cost of producing the backpacks includes a setup cost of $4500 and additional costs
of $30 per backpack.

1

    (a) Write the cost function in the form $y = mx + b$

    (b) Sketch the cost function on the set of axes below   1

Question 18 continued

(c) Hence, or otherwise, determine Sianne's break-even point.   **1**

　　　(i.e. determine how many backpacks she needs to sell to cover her costs?)

End of Question 18

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Question 19 (3 marks)

Find $\frac{d}{dx}\left(xe^x\right)$ and hence find $\int xe^x\,dx$ 3
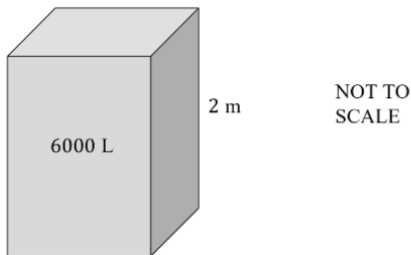
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Question 20 (4 marks)

A rainwater tank manufacturer wants to make a new tank that is lower than the **4** standard home fence height, to reduce the disturbance of views from neighbouring houses.

The manufacturer decides that the tank will be a 2 metre high rectangular prism, **excluding a roof**, and needs to hold 6000 litres of water.

6000 L   2 m   NOT TO SCALE

For environmental and commercial reasons the manufacturer wants to minimise the Surface Area of the tank i.e. use the least amount of material possible.

Calculate the exact dimensions of the tank that uses the smallest amount of material possible. (hint: $1000 \text{ L} = 1 \ m^3$)

There is more space to answer Question 20 on page 18

Question 20 more space if needed

End of Question 20

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Question 21 (6 marks)

There are three towns in a rural area. Town B is 37 km from town A at a bearing of 040°T.

From Town B , Town C is 45 km away and at a bearing of 160°T.

(a) Find the size of $\angle ABC$.   **2**

- 19 -

Office Use Only - Do NOT write anything, or make any mark below this line.

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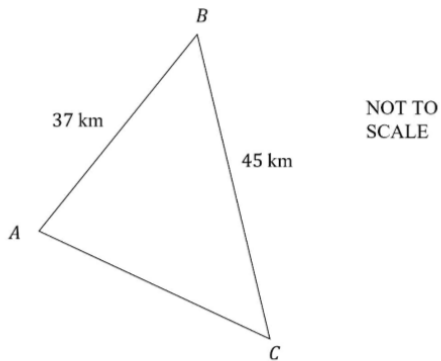
Question 21 (continued)

(b) Find the distance from town A to town C. Answer correct to 1 decimal place. **2**

(c) Find the bearing of town A from town C. **2**

End of Question 21

Question 22 (2 marks)

Ella has a bag that contains 5 red and 8 black balls.

Two balls are drawn randomly from the bag (without replacement)

    a)   Find the probability that the first ball drawn is red.   **1**

    b) Find the probability that both balls drawn are the same colour.   **1**

Question 23 (2 marks)

The table below shows the probability distribution of a discrete random variable X

x	0	2	4	5	8	9
P(X= x)	2k	0.16	0.18	0.3	k	0.12

1

a) Show that $k = 0.08$

b) Calculate $E(X)$

1

- 22 -

Office Use Only - Do NOT write anything, or make any mark below this line.

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Question 24 (3 marks)

(a) Simplify   2

$$\frac{\cos^2 \theta}{1 - \sin \theta} - \frac{\cos^2 \theta}{1 + \sin \theta}.$$
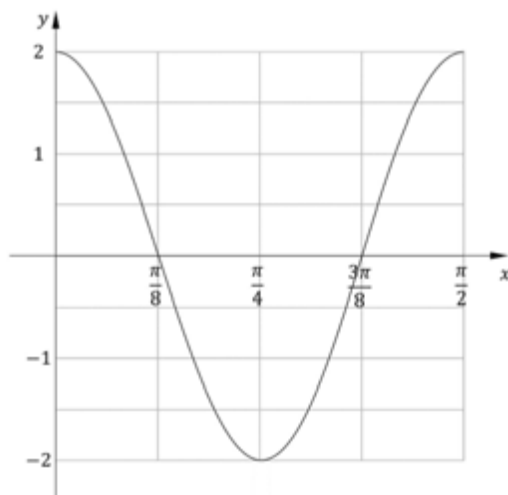
(b) Hence, solve   1

$$\frac{\cos^2 \theta}{1 - \sin \theta} - \frac{\cos^2 \theta}{1 + \sin \theta} = 1 \quad \text{for } 0 \leq \theta \leq \frac{\pi}{2}.$$

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Question 25 (4 marks)

The graph shown is $y = A \cos bx$.

(a) Find A   **1**

_____

_____

(b) Explain why $b = 4$.   **1**

_____

_____

_____

(c) On the axes above, draw the graph of $y = \sin 2x + 1$ for $0 \le x \le \frac{\pi}{2}$ **2**

_____

_____

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Question 26 (4 marks)

Solve: $2\sin^2\theta - \cos\theta = 1$ for $0 \le \theta \le 2\pi$.   **4**

Question 27 (6 marks)

(a) Find the turning points and classify them if $y = x^4 - 2x^3 + 1$.   **3**

b) Find any point(s) of inflection   **2**

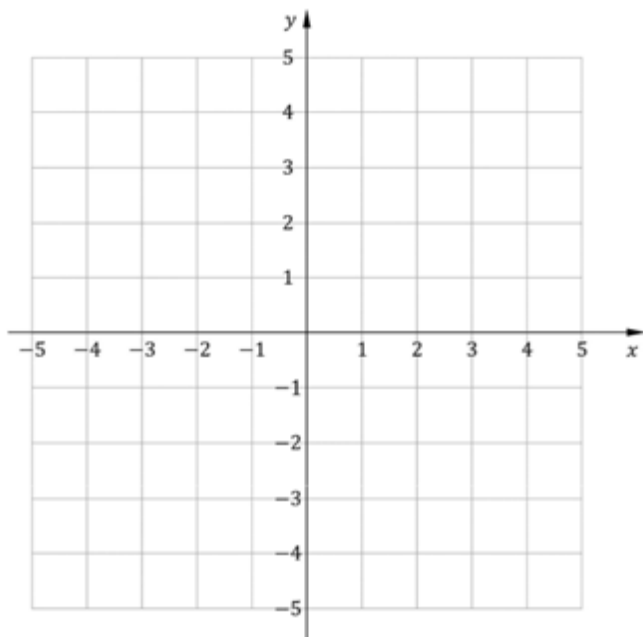**Question 27 continues on page 27**

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Question 27 (continued)

(b) Sketch the graph $y = x^4 - 2x^3 + 1$ on the axes below   **1**

    It is not necessary to find x-intercepts.

End of Question 27

Question 28 (3 marks)

The Moon has a lower gravity than Earth, and there is no atmosphere to cause air resistance, so a ball would bounce higher and for much longer on the Moon than on Earth.

3

When a ball is dropped on the Moon each bounce is 95% as high as the previous bounce. When an identical ball is dropped on Earth each bounce is 50% as high as the previous bounce.

NOT TO SCALE

2 metres

ball

Two identical balls are dropped on the Moon and on Earth, each from a height of 2 metres.

Calculate the difference in the **total** vertical distances travelled by these balls.

....

....

....

....

....

....

There is more space to answer Question 28 on page 29 if needed

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Question 28 (extra space if needed)
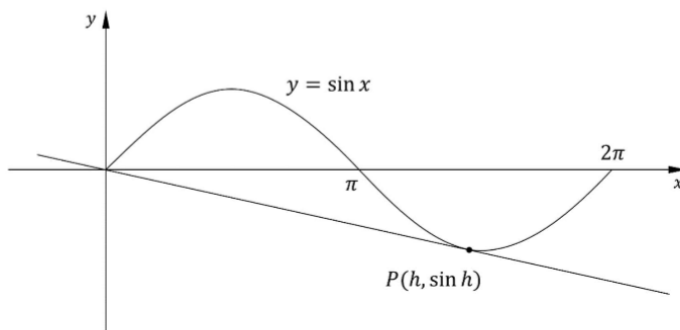
End of Question 28

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Question 29 (4 marks)

The graph of $y = \sin x$, shown below, has only one tangent in the domain $[\pi, 2\pi]$ **4**
that passes through the origin.

Let the point of contact of this tangent be $P(h, \sin h)$.

Prove that $h = \tan h$.

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Question 30 (6 marks)

A survey contained a question asking eight households to give their weekly income.

The results of the survey were as follows.

$600 $1200 $1600 $1800 $2000 $2400 $2600 $4200

(a) Calculate the 5-point summary (min, upper and lower quartiles, median, max)   **2**

(b) Are there any outliers – justify your answer by providing a suitable calculation   **2**

(c) A ninth household answers the survey. Their answer causes the median to increase and the mean to decrease.   **2**

Between what two values was their weekly income?

Question 31 (2 marks)

Find **2**

$$\frac{d}{dx}\left[\log_3\left(x^2\right)\right].$$

Question 32 (4 marks)

The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA can be modelled by the function $P(t) = 180 + 44\sin\left(\frac{2\pi t}{183}\right)$, where t is the number of days after 22 March 2023, for $0 \leq t \leq 366$.

    a)   What is the maximum price of petrol during the year   **1**

    b)   Sketch the function $P(t)$ for $0 \leq t \leq 366$.   **3**

Question 33 (4 marks)

The equation $f(x) = \cos\left(\frac{x}{2}\right)$ is a probability density function for $\left[\frac{\pi}{3}, \pi\right]$

(a) Find the cumulative distribution function in the given domain.　　　　**2**

(b) Hence, or otherwise, find the median of this data to two decimal places.　　　　**2**

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Question 34 (2 marks)

A researcher defined Scale A and Scale B to represent the magnitude of an explosion as shown in the table below:

2

It is given that E is the relative energy released by the explosion. If the magnitude N of an explosion is 6.2 on Scale B, find the magnitude M of the explosion on Scale A.

Scale	Formula
A	$M = \log_4 E$
B	$N = \log_8 E$

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Question 35 (7 marks)

The acceleration of a particle P_1 is $\dfrac{d^2x}{dt^2} = e^{-t} + e^{-2t}$ in $m|s^2$, after t seconds. Initially, the particle is $\dfrac{3}{4}m$ to the right of the origin, travelling at velocity $\dfrac{dx}{dt} = \dfrac{-3}{2}\ m/s$.

(a) Show that the displacement of the particle is given by

$$x = e^{-t} + \frac{1}{4}e^{-2t} - \frac{1}{2}$$

2

(b) Find the limiting displacement of P_1, and hence state the limiting distance that P_1 travels.

2

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Question 35 (continued)

Another particle, P_2, moves simultaneously with the first particle. The acceleration P_2 experiences is $\frac{d^2x}{dt^2} = -e^{-t} + e^{-2t}$ in m/s^2.   **3**

This particle is $\frac{3}{4} m$ to the right of the origin, travelling at velocity

$\frac{dx}{dt} = -\frac{3}{2}$ m/s, when $t = \ln 3$ seconds.

(c)   **Determine the exact time (after time t = 0) when P_1 and P_2, are travelling with the same velocity.**

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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SOLUTION.

Student Number

2023 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

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Section I - 10 marks (pages 2-6)

- Attempt Questions 1-10
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Section II - 90 marks (pages 7-40)

- Attempt Questions 11-34
- Allow about 2 hours 45 minutes for this section

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

1   Consider the bivariate data shown on the scatterplot below.

Which of the following values is the best estimate for Pearson's correlation coefficient for this data?

A. -0.9

B. -0.2

C. 0.2

D. 0.9

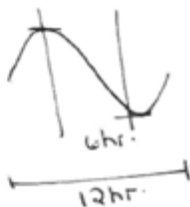2   The graph of the function $y = f(x)$ is moved 3 units to the left.

Which of the following is the new function?

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C. $y = f(x + 3)$

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3 The height of the tide in a harbour can be modelled using the sine function. The time, t in hours, between high tide and low tide is 6 hours.

Which of the following could be the function representing the height of the tide?

A. $h = \sin\left(\frac{\pi t}{3}\right)$

B. $h = \sin\left(\frac{\pi t}{6}\right)$

C. $h = \sin\left(\frac{\pi t}{12}\right)$

D. $h = \sin\left(\frac{\pi t}{18}\right)$

(handwritten)
6hr.
12hr.
$\text{period} = 12 = \frac{2\pi}{f}$
$12f = 2\pi$
$f = \frac{2\pi}{12} = \frac{\pi}{6}$

4 The graph of $y = f(x)$ is shown.

DRAWN
TO
SCALE

$y = f(x)$

What is the exact value of

$$\int_0^8 f(x)\, dx\ ?$$

A. $10 + \pi$ units^2

B. $10 - \pi$ units^2

C. $14 + \pi$ units^2

D. $14 - \pi$ units^2

(handwritten)
$A = \frac{1}{2}(2)(2) + (4)(2) + 4 - \frac{1}{4}\pi(2)^2$
$= 2 + 8 + 4 - \pi$
$= 14 - \pi$

5   Nadav scored 34 in his Science examination. The mean of the scores was 50, and
    Nadav's z-score in the Science examination was -2.
    What is the standard deviation of scores in the science examination?

    A.   4

    B.   8

    C.   12

    D.   16

$$x = 34$$
$$\bar{x} = 50$$
$$z = -2$$
$$z = \frac{x - \bar{x}}{\sigma}$$
$$-2 = \frac{34 - 50}{\sigma}$$
$$-2\sigma = -16$$
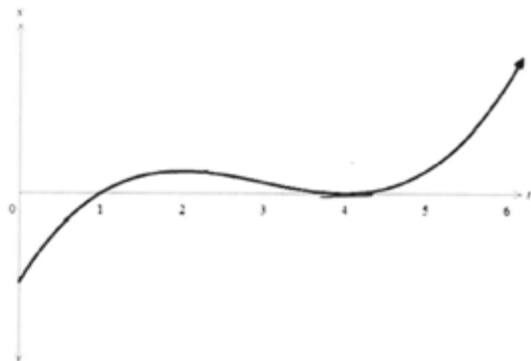
6   Consider the parallel box plots below. $\sigma = 8$

Group 1

Group 2

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21

Which of the following statements is CORRECT?

A.   Group 1 is positively skewed.

B.   Group 2 is negatively skewed.

C.   The difference between the median and Q_1 of Group 1 is the same as the
     difference between the median and Q_3 of Group 2.

D.   The range and IQR are equal for both sets of data.

7

The displacement, x metres, from the origin of a particle moving in a straight line at any time, t seconds, is shown in the graph below.

When was the particle at rest?

A. $t = 0$

B. $t = 1, t = 4$

C. $t = 2, \ t = 4$

D. $t = 1, \ t = 2, \ t = 4$

8   A standard normal distribution has a mean of zero and a standard deviation of 1.

Given that $P(z_1 < Z < z_2) = 0.815$, which of the following are possible values of z_1 and z_2?

A. $z_1 = -1, z_2 = 2$

B. $z_1 = -1, z_2 = 3$

C. $z_1 = -2, z_2 = 2$

D. $z_1 = -2, z_2 = 3$

9 What is the derivative of $\left(\tan^2 x + 1\right)^2$?

$y' = 2(\tan^2 x + 1)^1 \times 2\tan x \times \sec^2 x$

$= 4\sec^2 x \cdot \tan x \cdot \sec^2 x$

- A. $4\sec^3 x$
- B. $4\sec^4 x$
- C. $4\sec^3 x \tan x$
- D. $4\sec^4 x \tan x$

10 It is given that $P(A) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.

What is the value of $P(\bar{B}|\bar{A})$?

$P(\bar{B}/\bar{A}) = \dfrac{P(\bar{B} \cap \bar{A})}{P(\bar{A})}$

$= \dfrac{1 - P(A \cup B)}{1 - P(A)}$

$= \dfrac{1 - \frac{3}{5}}{1 - \frac{1}{2}} = \frac{4}{5}$

- A. $\frac{1}{4}$
- B. $\frac{3}{10}$
- C. $\frac{3}{4}$
- D. $\frac{4}{5}$

End of multiple-choice section

Mathematics Advanced

Section II Answer Booklet

Section II

90 marks
Attempt Questions 11–36
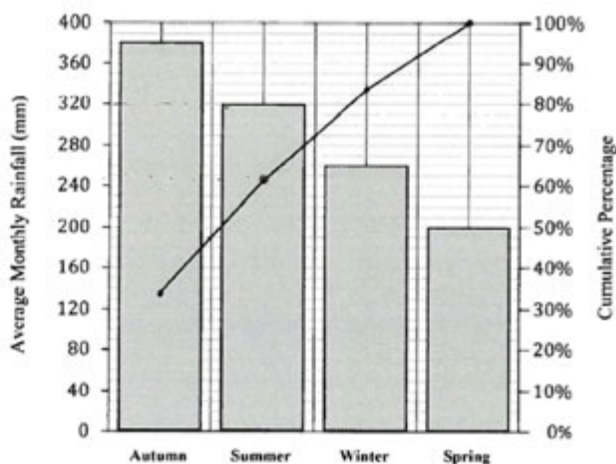Allow about 2 hours 45 minutes for this section

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Please turn over

Question 11 (3 marks)

Below is a Pareto Chart showing the average seasonal rainfall in Sydney.

(a) What is the average rainfall for Summer?　　　　　　　　　　　　1

　　　　320 mm

(b) What percentage of the annual rainfall occurs in Winter and Spring together?　　2

　　　　　100% — 60%　=　40%

　　　　　　　　(accept 38% — 42%)

Question 12 (2 marks)

Find

2

$$\frac{1}{2} \int \frac{21}{2x-3} \, dx.$$

$$= \frac{1}{2} \ln \left| (2x - 3) \right| + C$$

Question 13 (3 marks)

Calculate and fully simplify:

3

$$\int_{\ln 2}^{2\ln 2} e^{2x} \, dx.$$

$$\left[\frac{e^{2x}}{2} \right]_{\ln 2}^{2\ln 2}$$

$$\frac{1}{2} \left[e^{4\ln 2} - e^{2\ln 2} \right]$$

$$= \frac{1}{2} \left[e^{\ln 16} - e^{\ln 4} \right]$$

$$= \frac{1}{2} \left[16 - 4 \right]$$

$$= \frac{1}{2} \left[12 \right]$$

$$= 6.$$

Question 14 (3 marks)

It is given that $f''(x) = 6x$ and that $f(x)$ has a stationary point at $(-1,3)$. **3**

Find $f(x)$.

$$f'(x) = \int 6x \, dx$$

$$f'(x) = \frac{6x^2}{2} + c$$

$$\therefore f'(-1) = 3(-1)^2 + c = 0.$$

$$c = -3.$$

$$f'(x) = 3x^2 - 3$$

$$f(x) = \int 3x^2 - 3 \, dx$$

$$f(x) = \frac{3x^3}{3} - 3x + d$$

$$f(-1) = (-1)^3 - 3(-1) + d = 3$$

$$-1 + 3 + d = 3$$

$$d = 1$$

$$\therefore f(x) = x^3 - 3x + 1$$

Question 15 (3 marks)

(a) It is given that $1 + 2x - 3x^2 \geq 0$ in the domain $[0,1]$.

Prove that $f(x) = 1 + 2x - 3x^2$ is a probability density function for $[0,1]$.   **2**

$$A = \int_0^1 1 + 2x - 3x^2 \, dx$$

$$= \left[x + \frac{2x^2}{2} - \frac{3x^3}{3} \right]_0^1$$

$$= (1 + 1 - 1) - (0)$$

$$= 1.$$

since area $= 1$

PDF.

(b) State the mode of the probability density function.   **1**

mode $=$ highest value

$$\frac{d}{dx}(1 + 2x - 3x^2)$$

$$= 2 - 6x = 0$$

$$-6x = -2$$

$$x = \frac{1}{3}.$$

Question 16 (4 marks)

Find and fully factorise the first and second derivatives of $y = x(x+1)^3$.   4

$$u = x \qquad v = (x+1)^3$$
$$u' = 1 \qquad v' = 3(x+1)^2 \times 1$$

$$y' = (x+1)^3 + 3x(x+1)^2$$
$$y' = (x+1)^2(x+1+3x)$$
$$y' = (x+1)^2(4x+1)$$

$$u = (x+1)^2 \qquad v = 4x+1$$
$$u' = 2(x+1)^1 \qquad v' = 4$$

$$y'' = 2(x+1)(4x+1) + 4(x+1)^2$$
$$= (x+1)(8x+2+4x+4)$$
$$= (x+1)(12x+6)$$
$$= 2(x+1)(2x+1)$$

- 12 -

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Question 17 (3 marks)

The circle $x^2 + y^2 - 6x + 8y - 11 = 0$ is transformed by a horizontal translation   **3**
to the left by 4 units and a vertical translation up 3 units.

What is the centre and radius of the new circle?

$$x^2 - 6x + (3)^2 + y^2 + 8y + (4)^2 = 11 + 9 + 16$$

$$(x - 3)^2 + (y + 4)^2 = 36$$

$$\text{centre } (3, -4) \qquad r = 6$$

$\leftarrow$

$$\therefore \text{ new centre } (-1, -1) \qquad r = 6$$

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Question 18 (3 marks)

Sianne owns a company producing and selling backpacks.
The income is defined by the function $y = 80x$, where x is the number of backpacks sold.
The cost of producing the backpacks includes a setup cost of $4500 and additional costs
of $30 per backpack.

1

    (a) Write the cost function in the form $y = mx + b$

$$y = 30x + 4500$$

    (b) Sketch the cost function on the set of axes below
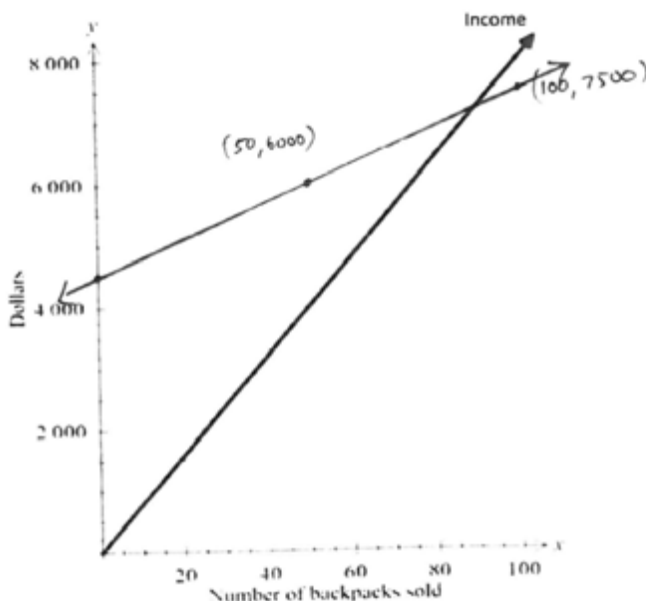
1

(c) Hence, or otherwise, determine Sianne's break-even point.

    (i.e. determine how many backpacks she needs to sell to cover her costs?)

1

$$80x = 30x + 4500$$
$$50x = 4500$$
$$x = 90.$$

needs to sell 90 backpacks to break even.

End of Question 18

Question 19 (3 marks)

Find $\frac{d}{dx}(xe^x)$ and hence find $\int xe^x\,dx$

3

$$u = x \qquad\qquad v = e^x$$
$$u' = 1 \qquad\qquad v' = e^x$$

$$y' = e^x + xe^x$$

$$\therefore \int e^x + xe^x\,dx = xe^x + c$$

$$\int e^x\,dx + \int xe^x\,dx = xe^x + c$$

$$e^x + \int xe^x\,dx = xe^x + c$$

$$\int xe^x\,dx = xe^x - e^x + c$$

Question 20 (4 marks)

A rainwater tank manufacturer wants to make a new tank that is lower than the
standard home fence height, to reduce the disturbance of views from neighbouring
houses.

4

The manufacturer decides that the tank will be a 2 metre high rectangular prism,
excluding a roof, and needs to hold 6000 litres of water.

6000 L

2 m

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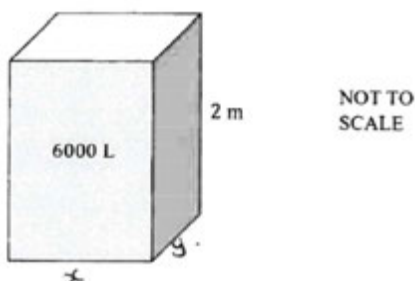
For environmental and commercial reasons the manufacturer wants to minimise
the Surface Area of the tank i.e. use the least amount of material possible.

Calculate the exact dimensions of the tank that uses the smallest amount of
material possible. (hint: $1000\ L = 1\ m^3$)

$$V = 2xy = 6$$
$$xy = 3$$
$$y = \frac{3}{x}$$

$$SA = 2(2x) + 2(2y) + xy$$
$$A = 4x + 4\left(\frac{3}{x}\right) + 3$$
$$A = 4x + 12x^{-1} + 3$$
$$\frac{dA}{dx} = 4 - 12x^{-2} = 0$$

There is more space to answer Question 20 on page 18

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Question 20 more space if needed

$$A = \frac{12}{x^2}$$

$$Ax^2 = 12$$

$$x^2 = 3$$

$$x = \sqrt{3}.$$

$$y = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}.$$

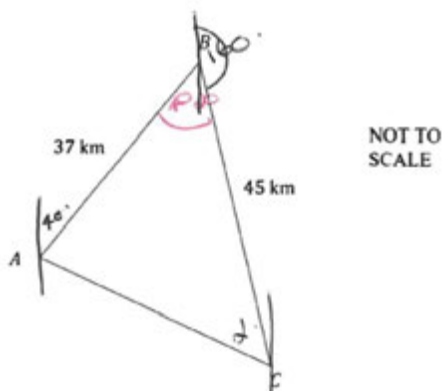$$\frac{d^2A}{dx^2} = 12x^{-3} = \frac{12}{x^3} > 0 \quad \vee \text{min}$$

dimensions $\sqrt{3} \times \sqrt{3} \times 2.$

End of Question 20

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Question 21 (6 marks)

There are three towns in a rural area. Town B is 37 km from town A at a bearing of 040°T.

From Town B, Town C is 45 km away and at a bearing of 160°T.

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(a) Find the size of $\angle ABC$.　　　　　　　　　　2

$\angle ABC = 60°$

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(b) Find the distance from town A to town C. Answer correct to 1 decimal place.   2

$$AC^2 = 37^2 + 45^2 - 2 \times 37 \times 45 \cos 60$$
$$AC^2 = 1729$$
$$AC = 41.6 \text{ km}$$

(c) Find the bearing of town A from town C.   2

$$\frac{\sin \alpha}{37} = \frac{\sin 60}{41.6}$$
$$\alpha = 50°25'$$

True bearing $= 289°35'$ T
$= 290°$ T (nearest degree)

Compass bearing $=$ N 70° W (nearest degree)
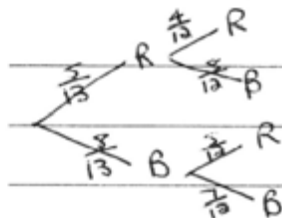
End of Question 21

Question 22 (2 marks)

Ella has a bag that contains 5 red and 8 black balls.

Two balls are drawn randomly from the bag (without replacement)

    a)   Find the probability that the first ball drawn is red.   **1**

$$P(\text{red}) = \frac{5}{13}.$$

    b) Find the probability that both balls drawn are the same colour.   **1**

$$P(RR) + P(BB)$$

$$= \frac{5}{13} \times \frac{4}{12} + \frac{8}{13} \times \frac{7}{12}$$

$$= \frac{19}{39}$$

- 21 -

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Question 23 (2 marks)

The table below shows the probability distribution of a discrete random variable X

x	0	2	4	5	8	9
P(X= x)	2k	0.16	0.18	0.3	k	0.12

1

a) Show that $k = 0.08$

$$2k + 0.16 + 0.18 + 0.3 + k + 0.12 = 1$$

$$3k = 1 - 0.76$$

$$3k = 0.24 \quad \checkmark$$

$$k = 0.08$$

b) Calculate $E(X)$

1

$$E(x) = \sum x \cdot px$$
$$= (0) + 2 \times 0.16 + 4 \times 0.18 + 5 \times 0.3$$
$$\quad + 8 \times 0.08 + 9 \times 0.12$$

$$= 4.26 \quad \checkmark$$

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Question 24 (3 marks)

(a) Simplify   **2**

$$\frac{\cos^2\theta}{1-\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta}.$$

$$\frac{\cos^2\theta(1+\sin\theta) - \cos^2\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} \checkmark$$

$$= \frac{\cos^2\theta + \cos^2\theta\sin\theta - \cos^2\theta + \cos^2\theta\sin\theta}{1-\sin^2\theta}$$

$$= \frac{2\cos^2\theta\sin\theta}{\cos^2\theta} \checkmark \qquad 2$$

$$= 2\sin\theta$$
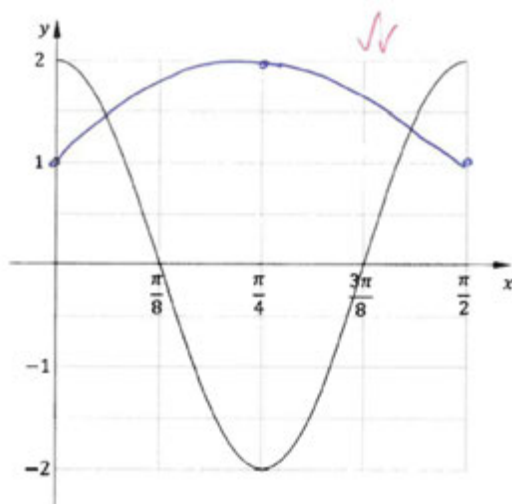
(b) Hence, solve   **1**

$$\frac{\cos^2\theta}{1-\sin\theta} - \frac{\cos^2\theta}{1+\sin\theta} = 1 \quad \text{for } 0 \le \theta \le \frac{\pi}{2}.$$

$$2\sin\theta = 1$$

$$\sin\theta = \frac{1}{2}$$

$$\text{rel} \angle = \frac{\pi}{6} \qquad 1.$$

$$\therefore \theta = \frac{\pi}{6} \checkmark$$

- 23 -

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Question 25 (4 marks)

The graph shown is $y = A \cos bx$.

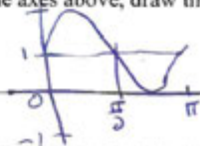(a) Find A   **1**

$A = 2.$ ✓

(b) Explain why $b = 4$.   **1**

, $freq = 4$

4 complete cos graphs ✓
between 0 & 2π.

(c) On the axes above, draw the graph of $y = \sin 2x + 1$ for $0 \le x \le \frac{\pi}{2}$ **2**

2

Question 26 (4 marks)

Solve: $2\sin^2\theta - \cos\theta = 1$ for $0 \le \theta \le 2\pi$.   **4**

$$2(1 - \cos^2\theta) - \cos\theta - 1 = 0$$

$$2 - 2\cos^2\theta - \cos\theta - 1 = 0$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$\cos\theta = \frac{1}{2} \qquad\qquad \cos\theta = -1$$

$$\text{rel } \alpha = \frac{\pi}{3}$$

1st	4th

$$\theta = \frac{\pi}{3} \qquad \theta = 2\pi - \frac{\pi}{3}$$

$$\theta = \frac{5\pi}{3} \qquad\qquad\qquad \theta = \pi$$

4

Question 27 (6 marks)

(a) Find the turning points and classify them if $y = x^4 - 2x^3 + 1$.   3

$$\text{S.P:} \quad y' = 0$$

$$y' = 4x^3 - 6x^2 = 0$$

$$2x^2(2x - 3) = 0 \quad \checkmark$$

$$x = 0 \qquad\qquad x = \frac{3}{2}$$

$$y = 1 \qquad\qquad y = \frac{-11}{16}$$

$$(0, 1) \quad \checkmark \qquad \left(\frac{3}{2}, \frac{-11}{16}\right) \checkmark$$

$$\text{HPOI} \checkmark \qquad\qquad\qquad \text{min}$$

$$y'' = 12x^2 - 12x$$

$$\text{at } x = 0. \qquad\qquad \text{at } x = \frac{3}{2}$$

$$y'' = 0. \qquad\qquad \quad 3 \qquad y'' = 9 \checkmark \text{ min.}$$

b) Find any point(s) of inflection   2

$$y'' = 0: \qquad 12x(x - 1) = 0.$$

$$x = 0 \checkmark \qquad x = 1 \checkmark$$
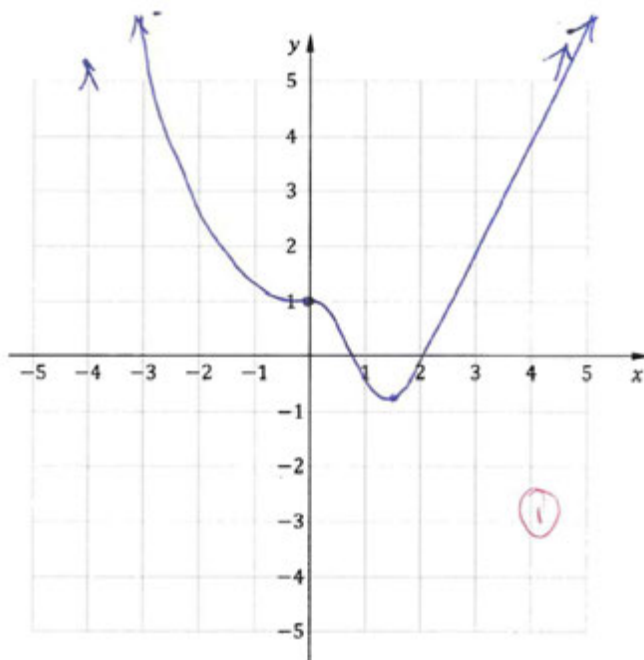
$$y = 1 \qquad\quad y = 0.$$

$$2.$$

Question 27 continues on page 27

7265152443

(b) Sketch the graph $y = x^4 - 2x^3 + 1$ on the axes below   1

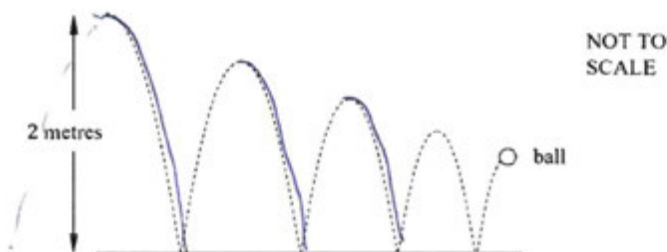   **It is not necessary to find x-intercepts.**

End of Question 27

Question 28 (3 marks)

The Moon has a lower gravity than Earth, and there is no atmosphere to cause air resistance, so a ball would bounce higher and for much longer on the Moon than on Earth.

3

When a ball is dropped on the Moon each bounce is <u>95%</u> as high as the previous bounce. When an identical ball is dropped on Earth each bounce is 50% as high as the previous bounce.

2 metres

NOT TO SCALE

ball

Two identical balls are dropped on the Moon and on Earth, each from a height of 2 metres.

Calculate the difference in the **total** vertical distances travelled by these balls.

$$\text{Moon} \qquad\qquad \text{Earth}$$

$$2, \ 95\% \times 2 \ \dots \qquad\qquad 2, \ 50\% \times 2 \ \dots$$

$$S_\infty = \frac{2}{1-95\%} \qquad\qquad S_\infty = \frac{2}{1-50\%}$$

$$= 40 \ \checkmark \qquad\qquad = 4 \cdot \checkmark$$

$$\therefore \ \text{total distance} \qquad\qquad \text{total distance} \quad ③$$

$$= 40 \times 2 - 2 \qquad\qquad 4 \times 2 \ -2$$

$$= 78 \cdot \ \checkmark \qquad\qquad = 6 \cdot \ \checkmark$$

There is more space to answer Question 28 on page 29 if needed

$$\therefore \ \text{diff} = 78 - 6 = 72 \ \checkmark$$

- 28 -

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Question 29 (4 marks)

The graph of $y = \sin x$, shown below, has only one tangent in the domain $[\pi, 2\pi]$ **4**
that passes through the origin.

Let the point of contact of this tangent be $P(h, \sin h)$.
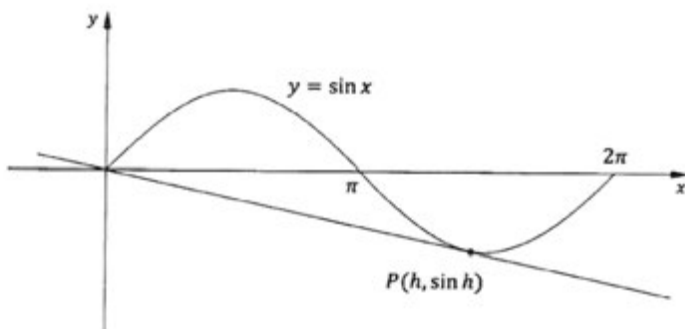
Prove that $h = \tan h$.

$m(\tan) = y' = \cos x$ ✓

at $x = h$

$m(\tan) = \cos h$ ✓

eq. of tangent thro' origin

$y - 0 = \cos h (x - 0)$ ④

$y = x \cos h$ ✓

But tangent $=$ sin curve at $x = h$.

$\dfrac{\sin h}{\cos h} = \dfrac{x \cos h}{\cos h}$

$\tan h = h$ ✓
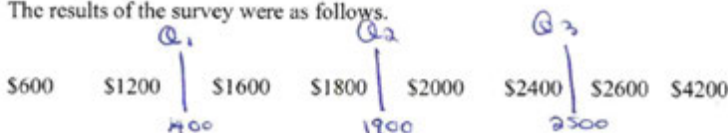
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$\therefore$ difference is $78 - 6 = 72\,m$.

End of Question 28

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Question 30 (6 marks)

A survey contained a question asking eight households to give their weekly income.

The results of the survey were as follows.

Q_1 Q_2 Q_3

$600 $1200 | $1600 $1800 | $2000 $2400 | $2600 $4200

1400   *1900*   *2500*

(a) Calculate the 5-point summary (min, upper and lower quartiles, median, max)   **2**

min 600

$Q_1 = 1400$

median $= 1900$

$Q_3 = 2500$

max $= 4200$.

(b) Are there any outliers – justify your answer by providing a suitable calculation   **2**

$IQR = Q_3 - Q_1 = 2500 - 1400 = 1100$

$\therefore 1\frac{1}{2} \times 1100 = 1650$.

$Q_3 + 1650 = 4150$ 4200 is an outlier

$Q_1 - 1650 = -250$ $\therefore$

(c) A ninth household answers the survey. Their answer causes the median to increase and the mean to decrease.   **2**

Between what two values was their weekly income?

> 1900 mean $= \dfrac{16400}{9}$

                             ≥ 2050

$\$1900 < x < \$2050$

- 31 -

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Question 31 (2 marks)

Find

2

$$\frac{d}{dx}\left[\log_3 \left(x^2\right) \right].$$

$$y = \log_3 x^2$$

$$y = 2\log_3 x$$

$$y' = 2 \cdot \frac{1}{\ln 3} \cdot \frac{1}{x}$$

$$y' = \frac{2}{x\ln 3}$$

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Question 33 (4 marks)

The price $P(t)$ in cents per litre of unleaded petrol during an average year in Broome WA can be modelled by the function $P(t) = 180 + 44 \sin\left(\frac{2\pi t}{183}\right)$, where t is the number of days after 22 March 2023, for $0 \leq t \leq 366$.
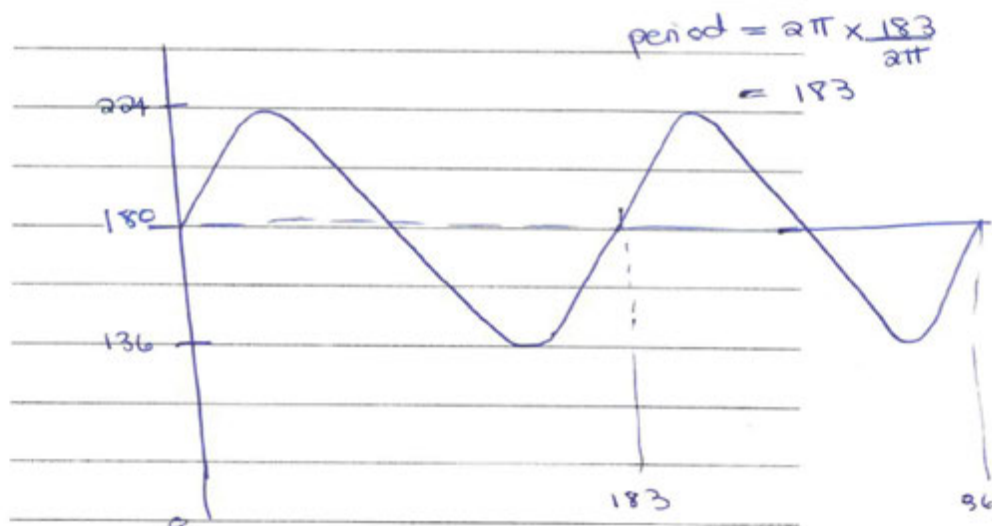
a) What is the maximum price of petrol during the year   1

max = 224.

b) Sketch the function $P(t)$ for $0 \leq t \leq 366$.   3

period $= 2\pi \times \dfrac{183}{2\pi}$

$= 183$

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Question 33 (4 marks)

The equation $f(x) = \cos\left(\frac{x}{2}\right)$ is a probability density function for $\left[\frac{\pi}{3}, \pi\right]$

(a) Find the cumulative distribution function in the given domain. **2**

$$CDF = \int_{\frac{\pi}{3}}^{x} \cos\left(\frac{x}{2}\right) dx$$

$-c \dfrac{-s}{-s} c$

$$= \left[\frac{\sin \frac{x}{2}}{\frac{1}{2}}\right]_{\frac{\pi}{3}}^{x}$$

$$= 2\left[\sin \frac{x}{2}\right]_{\frac{\pi}{3}}^{x}$$

$$= 2\left[\sin \frac{x}{2} - \sin \frac{\pi}{6}\right]$$

$$= 2\left[\sin \frac{x}{2} - \frac{1}{2}\right]$$

(b) Hence, or otherwise, find the median of this data to two decimal places. **2**

$$2\left[\sin \frac{x}{2} - \frac{1}{2}\right] = \frac{1}{2}$$

$$\sin \frac{x}{2} - \frac{1}{2} = \frac{1}{4}$$

$$\sin \frac{x}{2} = \frac{3}{4}$$

$$\text{ref } \lambda = 0.84$$

$$\frac{x}{2} = 0.848 \qquad \Bigg| \qquad \frac{x}{2} = \pi - 0.848$$

$$x = 1.70 \qquad \Bigg| \qquad x = \cancel{4.58}$$

$$\text{inad.}$$

- 34 -

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Question 34 (2 marks)

A researcher defined Scale A and Scale B to represent the magnitude of an explosion as shown in the table below:

2

It is given that E is the relative energy released by the explosion. If the magnitude N of an explosion is 6.2 on Scale B, find the magnitude M of the explosion on Scale A.

Scale	Formula
A	$M = \log_4 E$
B	$N = \log_8 E$

$$6.2 = \log_8 E$$
$$8^{6.2} = E$$

$$\therefore m = \log_4 8^{6.2}$$
$$= 6.2 \cdot \frac{\log 8}{\log 4}$$

$$= 9.3$$

Question 35 (7 marks)

The acceleration of a particle P_1 is $\frac{d^2x}{dt^2} = e^{-t} + e^{-2t}$ in $m|s^2$, after t seconds. Initially, the particle is $\frac{3}{4}m$ to the right of the origin, travelling at velocity $\frac{dx}{dt} = \frac{-3}{2}$ m/s.

(a) Show that the displacement of the particle is given by

$x = e^{-t} + \frac{1}{4}e^{-2t} - \frac{1}{2}$

$$a = e^{-t} + e^{-2t} \qquad\qquad t = 0$$

$$v = \int e^{-t} + e^{-2t} \, dt \qquad x = \frac{3}{4}$$

$$v = \frac{e^{-t}}{-1} + \frac{e^{-2t}}{-2} + c \qquad v = \frac{-3}{2}$$

$$-\frac{3}{2} = -e^{0} - \frac{1}{2}e^{0} + c$$

$$-\frac{3}{2} = -1 - \frac{1}{2} + c$$

$$0 = c$$

$$v = -e^{-t} - \frac{1}{2}e^{-2t}$$

$$x = \int -e^{-t} - \frac{1}{2}e^{-2t} \, dt$$

$$x = \frac{-e^{-t}}{-1} - \frac{1}{2}\frac{e^{-2t}}{-2} + d$$

$$\frac{3}{4} = e^{0} + \frac{1}{4}e^{0} + d$$

$$\frac{3}{4} = \frac{5}{4} + d \qquad\qquad \therefore d = -\frac{1}{2} \quad x = e^{-t} + \frac{1}{4}e^{-2t} - \frac{1}{2}$$

(b) Find the limiting displacement of P_1, and hence state the limiting distance that P_1 travels.

$$as\ t \to \infty \qquad\qquad \therefore distance$$

$$x \to -\frac{1}{2} \qquad\qquad \frac{3}{4} \to -\frac{1}{2} = \frac{5}{4}\,m$$

- 36 -

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Another particle, P_2, moves simultaneously with the first particle. The acceleration P_2 experiences is $\frac{d^2x}{dt^2} = -e^{-t} + e^{-2t}$ in $m|s^2$.

3

This particle is $\frac{3}{4}m$ to the right of the origin, travelling at velocity $\frac{dx}{dt} = -\frac{3}{2}$ m/s, when $t = \ln 3$ seconds.

(c) **Determine the exact time (after time t = 0) when P_1 and P_2, are travelling with the same velocity.**

$$a = -e^{-t} + e^{-2t}$$

$$v = -e^{-t} + e^{-2t} + c$$

$$-\frac{3}{2} = e^{-\ln 3} - \frac{1}{2}e^{-2\ln 3} + c$$

$$-\frac{3}{2} = e^{\ln \frac{1}{3}} - \frac{1}{2}e^{\ln \frac{1}{9}} + c$$

$$-\frac{3}{2} = \frac{1}{3} - \frac{1}{18} + c$$

$$-\frac{16}{9} = c$$

$$v = e^{-t} - \frac{1}{2}e^{-2t} - \frac{16}{9}$$

$$e^{-t} - \frac{1}{2}e^{-2t} - \frac{16}{9} = -e^{-t} - \frac{1}{2}e^{-2t}$$

$$2e^{-t} = \frac{16}{9}$$

$$\ln e^{-t} = \ln \frac{16}{18}$$

$t = \ln 3$

$x = \frac{3}{4}$

$v = -\frac{3}{2}$

$-t = \ln \frac{16}{18}$

$t = -\ln \frac{16}{18}$

End of paper

- 37 -

$t = 0.12$